Principles of Baseband Digital Data Transmission

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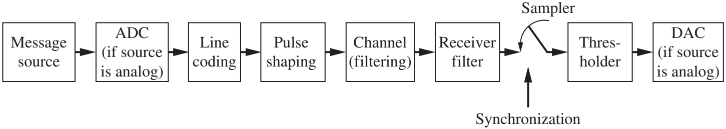
Overview

Baseband Digital Data Transmission Systems



- Analog-to-digital converter (ADC) block
  - ADC is required if the source produces an analog message
  - ADC consists of two operations
    1. Sampling
    2. Quantization
  - Quantization
    1. Rounding the samples to the nearest quantizing
    2. Converting them to a binary number representation
  - To avoid aliasing
    - Source had to be bandlimited to W Hz
    - Sampling rate had to satisfy $f_s > 2W$ samples per second (sps)
  - Signal is not bandlimited or $f_s < 2W \rightarrow$ **Aliasing** results
  - DAC (digital-to-analog converter) is the inverse operation of ADC

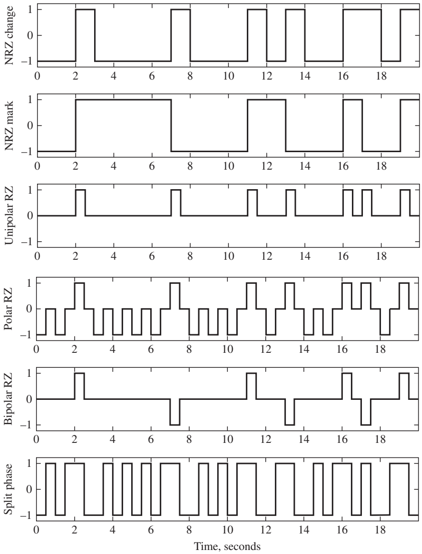
Baseband Digital Data Transmission Systems

- Line Coding
  - The purposes of line coding
    - Spectral shaping
    - Synchronization considerations
    - Bandwidth considerations
- Pulse shaping
  - Shaping the transmitted signal spectrum in order for it to be better accommodated by the transmission channel available
  - Severe degradation can result from transmitted pulses interfering with each other: **Intersymbol interference (ISI)**
  - We will see that careful selection of the combination of pulse shaping (transmitter and receiver filtering can completely eliminate ISI
- Synchronization
  - At the output of the receiver filter, it is necessary to synchronize the sampling times to coincide with the received pulse epochs
  - The samples of the received pulses are then compared with a threshold in order to make a decision as to whether a 0 or a 1 was sent

Line Codes and Their Power Spectra

- Description of Line Codes
  - The spectrum of a digitally modulated signal is influenced both by
    1. **Baseband data format** used to represent the digital data
    2. **Pulse shaping filtering** used to prepare the signal for transmission
  - Commonly used baseband data formats
    1. **Nonreturn-to-zero (NRZ)**: 1 is represented by a positive level A and 0 is represented by $-A$
    2. **NRZ mark**: 1 is represented by a change in level and 0 is represented by no change in level
    3. **Unipolar return-to-zero (RZ)**: 1 is represented by a $\frac{1}{2}$-width pulse and 0 is represented by no pulse
    4. **Polar RZ**: 1 is represented by a positive RZ pulse and 0 is represented by a negative RZ pulse
    5. **Bipolar RZ**: 0 is represented by a 0 level and 1s are represented by RZ pulses that alternate in sign
    6. **Split phase (Manchester)**: 1 is represented by A switching to $-A$ at $\frac{1}{2}$ the symbol period and 0 is represented by $-A$ switching to A at $\frac{1}{2}$ the symbol period

Line Codes and Their Power Spectra



Line Codes and Their Power Spectra

- Two of the most commonly used formats are **NRZ** and **split phase**
  - Split phase can be obtained from NRZ by multiplying it with a square wave clock waveform with a period equal to the symbol duration
- Considerations in choosing data formats
  1. Self-synchronization
  2. Power spectrum
  3. Transmission bandwidth
  4. Transparency
  5. Error detection capability
  6. Good bit error probability performance

Line Codes and Their Power Spectra

- Power Spectra for Line-Coded Data
  - A pulse-train signal

$$X(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT - \Delta) \tag{1}$$

where

- a_k is a sequence of r.v.s with the average $R_m = \langle a_k a_{k+m} \rangle, m = 0, \pm 1, \cdots$
- $p(t)$ is a deterministic pulse-type waveform
- Δ is a r.v. that is independent of the value of a_k and uniformly distributed in the interval $(-T/2, T/2)$

  - Autocorrelation function

$$R_X(\tau) = \sum_{m=-\infty}^{\infty} R_m r(\tau - mT) \tag{2}$$

where $r(\tau) = \frac{1}{T} \int_{-\infty}^{\infty} p(t+\tau)p(t)dt$

Line Codes and Their Power Spectra

- Power spectral density

$$S_X(f) = \mathcal{F}[R_X(\tau)] = \mathcal{F}\left[\sum_{m=-\infty}^{\infty} R_m r(\tau - mT)\right] \quad (3)$$

$$= \sum_{m=-\infty}^{\infty} R_m \mathcal{F}[r(\tau - mT)] \quad (4)$$

$$= \sum_{m=-\infty}^{\infty} R_m S_r(f) e^{-j2\pi mTf} \quad (5)$$

$$= S_r(f) \sum_{m=-\infty}^{\infty} R_m e^{-j2\pi mTf} \quad (6)$$

where

- $S_r(f) = \mathcal{F}[r(\tau)]$
- $r(\tau) = \frac{1}{T}\int_{-\infty}^{\infty} p(t+\tau)p(t)dt = \frac{1}{T}p(-t) * p(t) \rightarrow S_r(f) = \frac{|P(f)|^2}{T}$

Line Codes and Their Power Spectra

Example (PSD of a Split Phase Line Coding)

- Time average R_m

$$R_m = \begin{cases} \frac{1}{2}A^2 + \frac{1}{2}(-A)^2 = A^2 & \text{if } m = 0 \\ \frac{1}{4}A^2 - \frac{1}{4}A^2 - \frac{1}{4}A^2 + \frac{1}{4}A^2 = 0 & \text{if } m \neq 0 \end{cases} \quad (7)$$

- $p(t) = \prod\left(\frac{t+T/4}{T/2}\right) - \prod\left(\frac{t-T/4}{T/2}\right)$
- Fourier transform of $p(t)$

$$\begin{aligned} P(f) &= \frac{T}{2}\text{sinc}\left(\frac{T}{2}f\right)e^{j2\pi\frac{T}{4}f} - \frac{T}{2}\text{sinc}\left(\frac{T}{2}f\right)e^{-j2\pi\frac{T}{4}f} & (8) \\ &= \frac{T}{2}\text{sinc}\left(\frac{T}{2}f\right)(e^{j2\pi\frac{T}{4}f} - e^{-j2\pi\frac{T}{4}f}) & (9) \\ &= jT\text{sinc}\left(\frac{T}{2}f\right)\sin\left(\frac{\pi T}{2}f\right) & (10) \end{aligned}$$

Example (Continued)

Line Codes and Their Power Spectra

- From (10):

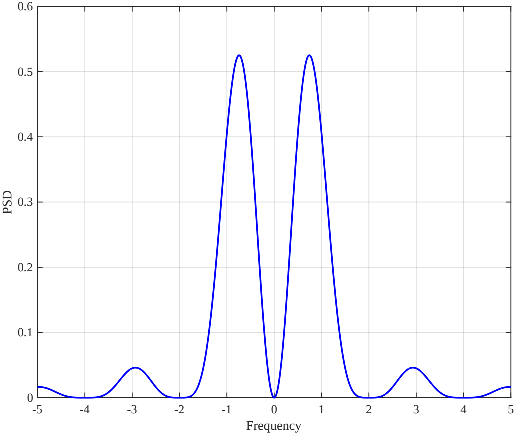
$$S_r(f) = \frac{1}{T}\left|jT\mathrm{sinc}\left(\frac{T}{2}f\right)\sin\left(\frac{\pi T}{2}f\right)\right|^2 \tag{11}$$

$$= T\mathrm{sinc}^2\left(\frac{T}{2}f\right)\sin^2\left(\frac{\pi T}{2}f\right) \tag{12}$$

- PSD of a split phase line coding

$$S_{\mathrm{SP}} = A^2T\mathrm{sinc}^2\left(\frac{T}{2}f\right)\sin^2\left(\frac{\pi T}{2}f\right) \tag{13}$$

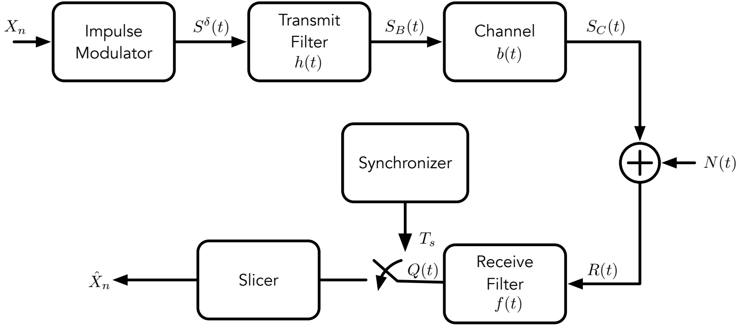
Line Codes and Their Power Spectra



Effect of Filtering of Digital Data: ISI

- One source of degradation in a digital data transmission is **inter-symbol interference (ISI)**
- ISI is induced when a sequence of signal pulses is passed through a channel with a bandwidth insufficient to pass the signal
- One of trivial transmit pulse shapes is a rectangular pulse with duration equal to the symbol duration
  - Spectrum of the transmitted signal is $|H(f)|^2$
  - Spectral characteristics of the rectangular pulse shape is undesirable
  - We wish to reduce the bandwidth required to transmit symbols
  - To reduce the required bandwidth $\rightarrow$ Increase the duration of $h(t)$
  - In order to obtain a bandlimited signal, the duration of $h(t)$ should be unlimited
  - The increase in the duration of $h(t)$ causes interference between transmitted symbols

Pulse Shaping Filter



Pulse Shaping Filter

- Let us assume for the time being that $b(t) = \delta(t)$ and $N(t) = 0$

$$
\begin{aligned}
Q(t) &= S_B(t) * f(t) && (14)\\
&= \left\{ \sum_{n=-\infty}^{\infty} X_n \delta(t - nT_s) * h(t) \right\} * f(t) && (15)\\
&= \sum_{n=-\infty}^{\infty} X_n h(t - nT_s) * f(t) && (16)\\
&= \sum_{n=-\infty}^{\infty} X_n p(t - nT_s) && (17)
\end{aligned}
$$

where $p(t) = h(t) * f(t)$ is called the overall pulse shape

Pulse Shaping Filter

- $Q(t)$ is sampled at the symbol rate to yield the decision variable Q_k

$$Q_k = Q(kT_s) = \sum_{n=-\infty}^{\infty} X_n p[(k-n)T_s] \quad (18)$$

$$= \sum_{n=-\infty}^{\infty} X_n p_{k-n} \quad (19)$$

where $p_l = p(lT_s)$

- We wish $Q_k = X_k$
- For $Q_k = X_k$, we require that $p_0 = 1$ **and** $p_l = 0$ **for** $l \neq 0$
- If the above condition is not satisfied, we will get

$$Q_k = X_k + \sum_{n \neq k} X_n P_{k-n} \quad (20)$$

$$= X_k + I_k \quad (21)$$

where I_k is called **inter-symbol interference (ISI)**

Pulse Shaping Filter

- The condition $p_l = \delta_l$ is called the condition for zero ISI or the **Nyquist condition (for zero ISI)**
- The overall pulse shape $p(t)$ satisfying the zero ISI condition is called **Nyquist pulse**
- Cleary, we would like to design $p(t)$ to be a Nyquist pulse

Pulse Shaping Filter

- The interpretation of the Nyquist condition in the frequency domain
  - If $p(t)$ satisfies the Nyquist condition:

$$p(t) \sum_{k=-\infty}^{\infty} \delta(t - kT_s) = \delta(t) \tag{22}$$

  - FT of the both side of (22)

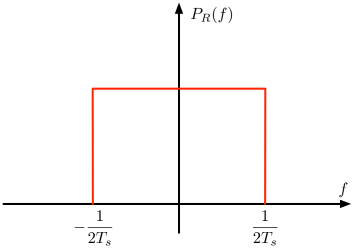
$$P(f) * \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \delta\left(f - k\frac{1}{T_s}\right) = 1 \tag{23}$$

$$\frac{1}{T_s} \sum_{k=-\infty}^{\infty} P\left(f - k\frac{1}{T_s}\right) = 1 \tag{24}$$

  - To satisfy the Nyquist condition, the **folded spectrum** $\sum_{k=-\infty}^{\infty} P\left(f - k\frac{1}{T_s}\right)$ must be constant!

Pulse Shaping Filter

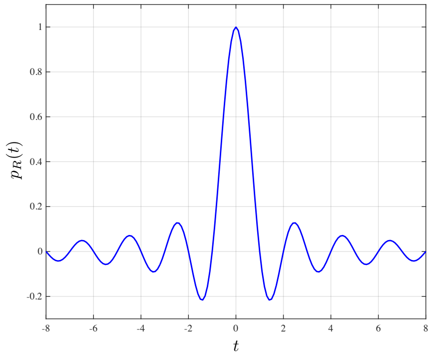
- Raised Cosine Filter
  - What is the **minimum bandwidth** required to transmit symbols at the rate of $\frac{1}{T_s}$ without ISI?



  - $P_R(f)$ satisfies the Nyquist condition and there can be no pulse shape with a smaller bandwidth that does
  - Corresponding time-domain pulse shape

$$p_R(t) = \frac{\sin\left(\frac{\pi t}{T_s}\right)}{\frac{\pi t}{T_s}} = \text{sinc}\left(\frac{t}{T_s}\right) \tag{25}$$

Pulse Shaping Filter



- Why isn't $p_R(t)$ universally employed?
  1. Impossible to obtain symbol synchronization
  2. Infinite sensitivity to timing error

Pulse Shaping Filter

- Consider $Q(\Delta)$
  - Consider a data pattern $X_0 = 1, X_{\pm 1} = -1, X_{\pm 2} = +1, X_{\pm 3} = -1, \cdots$

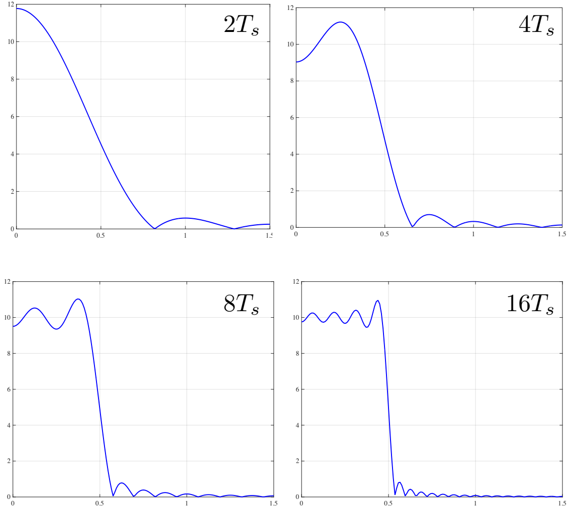
$$Q(\Delta) = X_0 p(\Delta) + \sum_{n \neq 0} X_n p(nT_s + \Delta) \quad (26)$$

$$= X_0 \mathrm{sinc}(\Delta) + \underbrace{\sum_{n \neq 0} X_n \mathrm{sinc}(nT_s + \Delta)}_{=I} \quad (27)$$

  - Note that the envelope of $\mathrm{sinc}(t)$ decrease only linearly in t
  - The term I will not converge and result in $Q(\Delta) = -\infty$ for any $\Delta > 0$

Pulse Shaping Filter

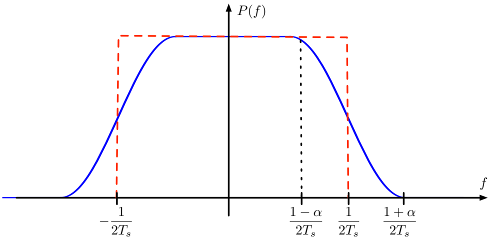
- One solution to this problem is to truncate the sinc function to have a finite duration



Pulse Shaping Filter

- The spectrum of the truncated sinc pulse is simply the convolution of sinc pulse and a rectangular pulse
- This example shows that we need a better or rather practical design for $p(t)$

- The Raised Cosine Filter
  - We observed that the truncated sinc pulse is not a desirable approach
  - We allow an **excess bandwidth** of $\frac{\alpha}{2T_s}$ Hz beyond $\frac{1}{2T_s}$
  - Transition band from $\frac{1-\alpha}{2T_s}$ to $\frac{1+\alpha}{2T_s}$
  - The transition band is designed to satisfy the **Nyquist condition**



  - The factor α is called the **roll-off factor**

Pulse Shaping Filter

- The transition band must be symmetric about $\frac{1}{2T_s}$ $\rightarrow$ Nyquist condition is satisfied!
- This will also result in a **bandlimited** pulse which is **not time limited**!
- We will obtain a pulse with an envelope that decreases faster than the sinc pulse which will allow us to truncate the pulse with a shorter duration window
- The DeFacto standard for the overall pulse shape is the **Raise Cosine (RC)**
- An RC pulse with a roll-off factor α

$$
\begin{aligned}
p_{RC}(t) &= \frac{\sin\left(\frac{\pi t}{T_s}\right)}{\frac{\pi t}{T_s}} \left(\frac{\cos\left(\frac{\alpha \pi t}{T_s}\right)}{1 - \left(\frac{2\alpha t}{T_s}\right)^2} \right) & (28) \\
&= \text{sinc}\left(\frac{t}{T_s}\right) m(t) & (29)
\end{aligned}
$$

- Note that $p_{RC}(t) = \text{sinc}\left(\frac{t}{T_s}\right) g(t)$ satisfies the Nyquist condition for any weighting function $g(t)$

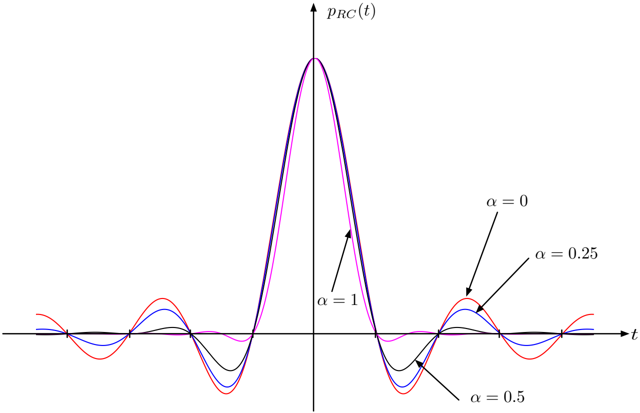
Pulse Shaping Filter

- RC filter is just a special case where $g(t) = m(t)$
- The envelope of the RC pulse decreases inverse linearly in t^3 as opposed to t for the sinc pulse
- Clearly, it is desirable that the transition band be smooth so as to allow practical filter design
- The spectrum of $p_{RC}(t)$

$$P_{RC}(f) = \begin{cases} T_s & \text{if } 0 \le |f| \le \frac{1-\alpha}{2T_s} \\ \frac{T_s}{2}\left\{1 + \cos\left[\frac{\pi T_s}{\alpha}\left(|f| - \frac{1-\alpha}{2T_s}\right)\right]\right\} & \text{if } \frac{1-\alpha}{2T_s} \le |f| \le \frac{1+\alpha}{2T_s} \\ 0 & \text{if } |f| > \frac{1+\alpha}{2T_s} \end{cases} \tag{30}$$

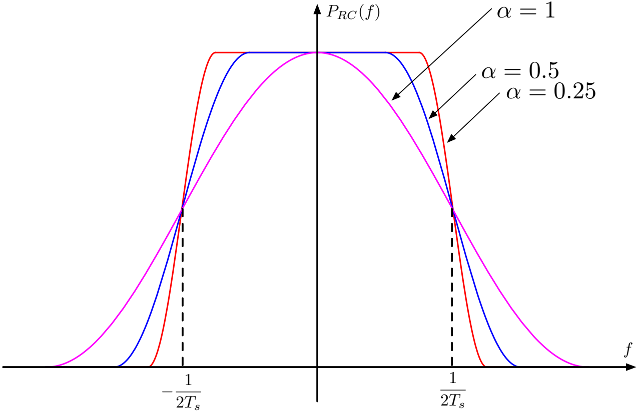
Pulse Shaping Filter

- Raised cosine pulses versus roll-off factor α



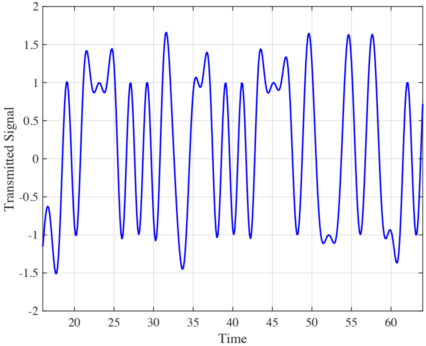
Pulse Shaping Filter

- Spectrum of raised cosine pulses versus roll-off factor α



Pulse Shaping Filter

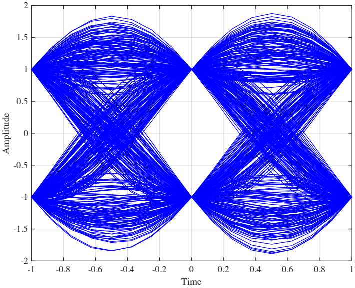
- Eye Diagram
  - Transmitted signal with the RC filter: Roll-off factor=0.25



  - The above plot gives no idea as to the amount of ISI in the signal
  - **Eye diagram**
    1. Useful in evaluating the performance of a digital communication systems
    2. Especially useful in getting a first-hand idea of the amount of ISI in a given system

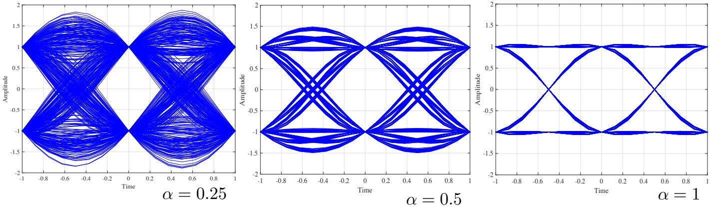
Pulse Shaping Filter

- The amount of ISI can easily be seen by folding this plot every nT_s seconds and overlapping the plots
- If the plot contains data for a large number of symbols, it will contain most of the possible transition paths
- Such a plot is called the **eye diagram**
- Eye diagram: $X_n \in \{-1, 1\}$ and a RC pulse of roll-off factor 0.25



Pulse Shaping Filter

- Eye diagrams versus roll-off factors



  - The Nyquist condition is exactly met and there is no ISI at the sampling points
  - The width of the eye is wider for larger roll-off factors
  - This makes the system less sensitive to timing (sampling time) errors
  - For $\alpha = 1$, we see well defined level (e.g. zero) between eyes which can be exploited to achieve timing synchronization